

THE ROLE OF SMALL-SCALE PHYSICAL MODELLING OF FLUID DYNAMIC PROCESSES IN MINERAL PROCESSING

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ABSTRACT

Fluid dynamic processes have been studied using small-scale physical models for many years. In particular, the aeronautics industry has made extensive use of wind tunnels and small-scale models to study the complex flows around aircraft, while the motor vehicle industry utilises a similar approach to study the aerodynamics of vehicles and their cooling systems.

Since the 1950s, small-scale physical models and wind tunnels have also been used to study the flows around buildings, and in the 1960s and 70s the Advanced Fluid Dynamics Laboratory (AFDL) of the CSIRO used small-scale physical modelling to develop fans and diffusers for mine ventilation systems.

Recently, the AFDL physically modelled a range of mineral processing equipment to solve significant problems encountered in the alumina industry. For these cases, single-phase liquids were used to model slurries in mixing tanks, while water was used to model the flows in an electrostatic precipitator. Slurries have also been developed which are optically clear, with the liquids and the solids having the same refractive indices.

Using experience gained by the researchers and computational fluid dynamics, the small-scale physical model results were scaled up to plant size and successfully trialed in the full-size installations. The reduced operational costs after modifying the plants in accordance with the recommendations of the small-scale physical modelling are substantial.

Successful collaboration on a project between industry and CSIRO is critically dependent on there being close communications between both parties throughout the project.

This paper has a companion video which illustrates the small-scale physical modelling techniques used at the AFDL.

KEY WORDS:

Small-scale Physical Modelling Agitation Mixing

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1.0 INTRODUCTION

Small-scale physical modelling of fluid dynamic processes is used extensively by the aeronautics industry. In particular, up until the 1970s it was used exclusively to develop new aircraft designs. Highly accurate models were constructed and located in wind tunnels to observe the complex flows around aircraft models while quantitative information was obtained using drag balances and surface pressure measurements.

This methodology was extended to the study of flow around road vehicles in Germany in the 1930s. Since that time, major vehicle manufacturers have developed extensive wind tunnel facilities and used small-scale physical models to study not only the aerodynamics of the flows around vehicles, but also the cooling systems and the generation of aerodynamic noise.

Small-scale physical modelling is also playing a major role in wind engineering where the flows around model buildings and over terrain have been studied using wind tunnels since the 1950s. This field has an additional complication in that the structures may be forced to vibrate by the wind and this vibration can 'feed back' and change the flows.

During the 1960s and 70s, the Advanced Fluid Dynamics Laboratory (AFDL) at CSIRO conducted many small-scale physical experiments on mine ventilation systems. These studies included the manufacture of and flow visualisation in 1:6 scale models of the 6 m diameter up and down cast fans and diffuser systems installed at Mt Isa mines; they also included a rationalisation of the techniques used to design axial flow fans (Wallis, 1983).

More recently, small-scale physical modelling was used at the AFDL to increase the performance and efficiency of equipment used in mineral processing refineries. Many of the fluids encountered in mineral processing have two or three phases and are shear thinning or shear thickening, thus requiring many more variables to express their make-up than do single-phase fluids such as water or air (Pullum *et al.*, 1994). In spite of the additional complexities of the fluids encountered in mineral processing, work at the AFDL showed that small-scale physical models can capture the essential features of these flows and can be used, together with computational fluid dynamics, to solve major process problems encountered in the mineral processing industry.

This paper describes the approach adopted by the AFDL to solve four different problems encountered in the alumina industry over the past three years using small-scale physical modelling and computational fluid dynamics.

2.0 NOTATION

a	speed of sound	m/s
D	rotor diameter	m
f	characteristic body force	N
l	characteristic length	m
N	rotor speed	rev s ⁻¹
v	velocity	m s ⁻¹
V_{∞}	particle setting velocity	m s ⁻¹
P	agitator power consumption	W
ρ	density	kg m ⁻³
μ	viscosity	Pa.s
μ_{app}	apparent viscosity for a given shear rate	Pa.s

Froude number ($F = v^2/lf$)

Generalised Reynolds number ($Re' = \rho v l / \mu_{app}$)

Mach number ($M = v/a$)

Power number ($P_N = P / \rho N^3 D^5$)

Reynolds number ($Re = \rho v l / \mu$)

3.0 METHODOLOGY AND PROCEDURE

3.1 General

It is well known that for complete similitude of single-phase, isothermal, compressible, viscous Newtonian flows around or in full-size and scale model equipment, it is necessary to maintain the same Froude, Reynolds and Mach numbers (Duncan *et al.*, 1960). This is not physically possible but often it is found that compressibility and body force effects can be ignored and it is only necessary to maintain *similar* Reynolds numbers for small-scale models to capture the essential features of the flows observed in a full-size installation.

3.2 Modelling Suspensions

As for single-phase Newtonian flows, in all cases, the small-scale physical modelling cannot capture all the features of the fluid dynamics of the full-size processes. However, it is possible to keep some of the non-dimensional groupings of the fluid dynamic and geometric variables the same as that observed in the full-size plant and therefore maintain some of the essential features of the flows in the small-scale models. Often, it is experience gained by trial and error which permits the researcher to determine those non-dimensional groupings which must be maintained constant, and those which can be ignored, while still successfully capturing the essential flow features of a full-size plant in a small-scale physical model.

Experience has shown that for small-scale modelling to be successful in modelling the fluid dynamics of mineral processing, it is essential to maintain geometric similarity and to match the apparent viscosity versus shear rate of the working fluid. In this way, homogenous slurry flows can be modelled by maintaining the generalised Reynolds number Re' in a similar manner to that for Newtonian flows. A more detailed non-dimensional analysis of some aspects of the fluid dynamics of suspensions is included in Appendix A

When the solids and fluids cannot be considered to be homogenous then other groups or properties have to be maintained to ensure similar behaviour. Our experience has shown that:

- the bulk suspension properties should always be modelled where possible, i.e. the volumetric concentration, or mass flow ratio in the case of pneumatic systems, and the solid/fluid density ratio should be maintained in the model;

- particle/fluid interaction can be characterised by the particle's settling velocity V_{∞} , and in the same way where particles interact with moving boundaries similarity can be achieved by maintaining the ratio between the boundaries velocity and V_{∞} ;
- the influence of external body forces, in particular gravity, on the suspension may be characterised with some form of Froude number; and
- if the particles are large compared to the vessel's dimensions, e.g. $>1:50$, then particle/vessel diameter ratios should also be maintained.

3.3 Procedure Adopted at the AFDL

The procedure adopted by the AFDL when investigating a fluid flow problem in the mineral processing industry is illustrated in Figure 1. For this particular case (a mixing tank), the first step is to determine the rheology of the fluids under investigation at the pressures and temperatures existing in the process. It is then necessary to develop a clear model working fluid which has the same apparent viscosity versus shear rate characteristic as the slurry measured on site. If the slurry is non-Newtonian, this can involve using a polymer and adjusting the concentration and the pH of the solution to match the rheology of the slurry. If the suspension is heterogeneous it can also involve developing a suspension consisting of a clear liquid and solid with the same refractive indices and rheology as that of the slurry. Flow visualisation and measurements can then be undertaken using the model suspension.

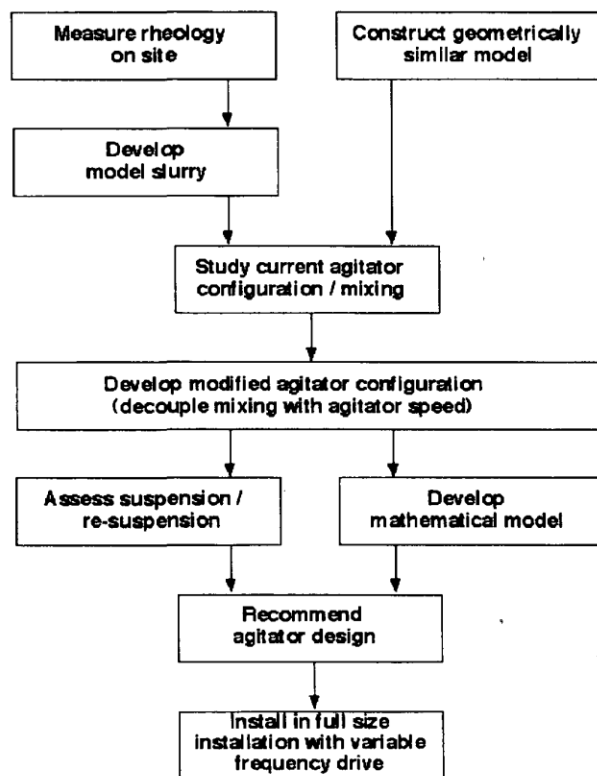


Figure 1
Example methodology adopted to solve fluid dynamic problems in the mineral processing industry at the AFDL (in this case for a mixing tank)

In general, the existing installation is physically modelled and the surface flows, power numbers and the residence times for the full-size installation and the model are compared to give confidence that the model is capturing the essential features of the full-size system.

After gaining confidence in the model, it can then be used to design a new agitator system to produce a flow pattern and subsequent mixing which are both very insensitive to the agitator speed of rotation. Decoupling the agitator speed from the mixing is a distinct advantage when scaling the agitator system and the agitator speed back up to the full size. A geometrically similar agitator can be installed in the full-size vessel with a variable frequency drive and then the speed can be progressively reduced while monitoring the concentration profile, for example, knowing that the flow patterns will be unchanged.

In some instances it may be necessary, and indeed most cost effective, to develop a mathematical model of the process using the data obtained from the small-scale physical model. Having used the small-scale physical model results to provide a limited validation of the mathematical model, the latter model can then be used to scale the results up from the small test rig up to the full-size plant.

4.0 FOUR EXAMPLES OF SUCCESSFUL SMALL-SCALE PHYSICAL MODELLING

4.1 General

The methods and procedures described above are general and it is normal to fine tune the process to achieve the solution to problems. Four recent examples of problems encountered in the alumina industry where small-scale physical modelling has played a major role in their solutions are:

- prevention of scale build-up in slurry tanks;
- design of a rotor for cage mills;
- prevention of massive scale build-up in digesters; and
- major increases in the collection efficiency of electrostatic precipitators.

For these cases, the slurry in the tank was modelled with a clear single-phase polymer liquid, the suspension in the cage mill was modelled using an air/solid suspension, the slurry in the digesters was modelled using a clear single-phase liquid, and the flows in the electrostatic precipitator were modelled using water.

The methodology used to study these problems and the benefits of their solutions to the companies are described below.

4.2 Prevention of Scale Build-up in Slurry Tanks

In this case, scale build-up on the inside surface of the tanks led to a 40% reduction in the volume of the tank over a six-month period. The company had received advice that the problem could be solved by increasing the power consumption of the agitators from 110 kW to 400 kW; this was a very expensive solution. Instead, a flow visualisation study was undertaken in a test rig at the AFDL which represented a 1:8.41 geometric scale model of the full-size tank and agitator system. The rheology of the slurry in the tanks was shown to be shear thinning with a yield stress. Since suspension and re-suspension of the solids in the slurry were unlikely to cause problems, because of the high viscosity of the slurry, the rheology of the slurry was simulated using a clear single-phase polymer solution having the same yield stress and shear thinning characteristics.

Initial studies using the existing agitator showed regions of stagnant fluid corresponding to the shape of the scale build-up in the full-size tanks. It was therefore hypothesised that the scale build-up on the inside surface of the tanks could be reduced if stagnant regions of fluid were eliminated. Also, the power numbers of the model agitator of the existing system agreed well with the measured power numbers in the full-size installation. This data gave confidence that the physical model was simulating the macro flow structures in the full-size tank.

Close observation of the flow in the model tank showed that the rotors on the proposed agitator were generating radial flows rather than axial flows, i.e. the rotors were stalled and were required to do more work than they were capable of doing. Additional flow visualisation studies showed that the stagnant regions could be removed simply by relocating the rotors on the central drive shaft; this adjustment led to a 25% reduction in the power consumption of the agitator. When the full-size agitator system was modified in accordance with the recommendations of the study the following were observed:

- scale growth was reduced to less than 5% of the volume of the tank over an 12-month period;
- the power consumption of the agitator was reduced by 24%;
- the torque required to operate the agitator was reduced;
- the wear rate of the agitator was reduced significantly;
- increased on-line equipment availability; and
- operating costs reduced significantly.

Further details of this study can be found in Pullum *et al.* (1994).

The cost of the small-scale physical modelling to the company was \$65K and the benefits to the company were reduced operating costs of hundreds of thousands of dollars per tank per year.

4.3 Design of a Rotor for Cage Mills

This study was undertaken by the AFDL for a company wishing to modify an existing cage mill which is used as part of the alumina refining process. The cage mill is located below an agglomeration bath at the base of a vertical riser supplying feed stock to a bank of cyclones. The large, wide distribution produced by the agglomerate requires that some form of particle size control is employed, while limitations in the overall conveying air in the plant also require that the solids are accelerated to prevent particle deposition within the pneumatic riser. With the existing design the particle attrition was too severe, producing too small a feed stock, while the mill introduced instabilities into the plant.

In this case, a 1:4 scale model of the cage mill was constructed from acrylic and it was decided to use air, at ambient temperature and pressure, and polystyrene beads as the working fluid. With this model it was possible to maintain *exact* similarity between the full-size mill and the model mill for the following variables:

- duct Reynolds number;
- particle Reynolds number; and
- particle settling to peripheral velocity ratio.

In addition, it was possible to maintain *approximate* similarity between the full-size mill and the model mill for the following variables:

- density ratio of the solids and the gas; and
- particle diameter to duct diameter ratio.

Flow visualisation showed several shortcomings of the solids flow with the existing rotor, while tufting with goose feathers of the blades showed regions of separated flow. This data used in conjunction with a mathematical model of the solids flow in the rotor suggested a new rotor design which would maximise large particle attrition, minimise small particle attrition and reduce overall wear. This rotor design was subsequently constructed and tested in the model test rig and then implemented in the full-size plant where it has operated reliably since its installation two months ago.

4.4 Prevention of Massive Scale Build-up in Digesters

In some versions of the Bayer process, bauxite is digested in vertical pressure vessels divided into stages, each with a rotor on a common shaft. Scale build-up in digesters, used in the alumina industry is a major problem, particularly on the walls of the digester. The company commissioned the AFDL to investigate this problem and provide design modifications aimed at reducing the scale build-up to a minimum.

For this study, the rheology of the slurry was measured on site at the operating temperature and pressure. Suspension of the solids in the slurry was considered not to be a problem. Consequently, it was decided to develop a clear single-phase working fluid with the same apparent viscosity versus shear rate at ambient conditions as that of the slurry under operational conditions.

A 1:8 scale model (Figure 2) of the digester was constructed from acrylic with metallic scale model impellers. In the full-size vessel the scale growth on the wall was a minimum in the plane of a rotor and it was therefore hypothesised that if the flow velocity in this plane was maintained near the wall throughout the vessel, the rate of scale growth would be considerably reduced. Consequently, the flow velocity was measured near the wall in the plane of the rotor in the model test rig and then this velocity was used as the design value to minimise scale growth. This required a complete redesign of the agitators and internal features on the digester and subsequent validation in the scale model test rig and then in the full-size plant.

After a six-month trial period in a modified digester in the refinery, there was:

- increased on-line equipment availability.
- reduced vibration from the agitator;
- 65% less scale than was observed previously;
- 60% less power consumption by the impellers;
- higher average residence time distribution over the six-month period;
- less quartz attack;
- 50% reduction in the time taken to descale the vessel; and
- increased on-line equipment availability.

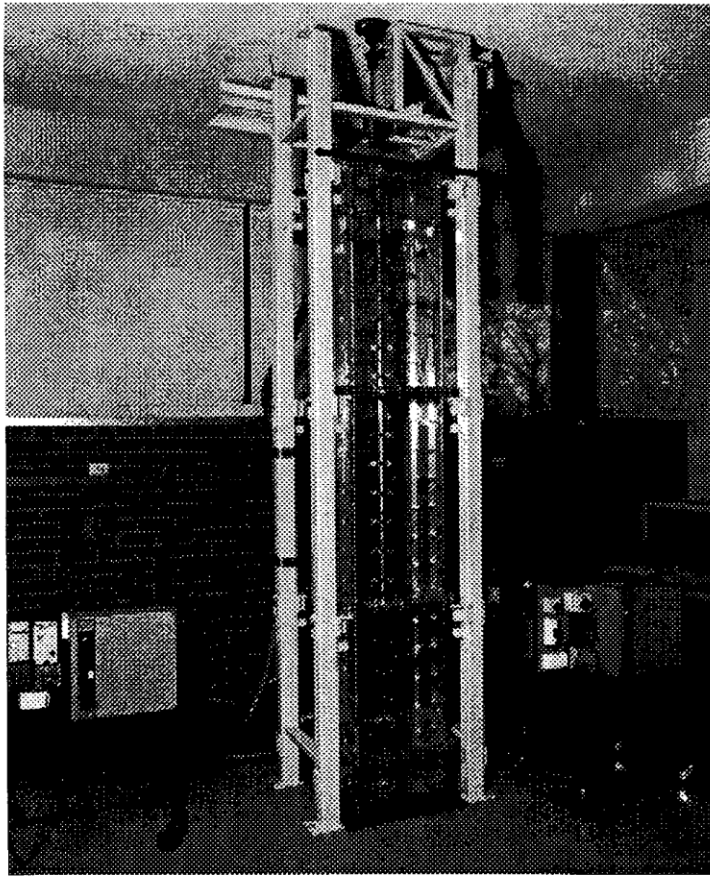


Figure 2
Photograph of the 1:8 scale model of the digester

The cost to the company of the small-scale physical modelling was approximately \$90K and the estimated saving to the company is hundreds of thousands of dollars per year per vessel, including process benefits.

4.5 Major Increases in the Collection Efficiency of Electrostatic Precipitators

An alumina company had experienced operational difficulties with an electrostatic precipitator (ESP) both before and after a major upgrade. Several of the modifications made during the upgrade were designed to improve the gas flow distribution through the ESP. However, exhaust stack particulate emissions remained highly variable after the upgrade and continued to fall well outside design limits.

It was thought that the flow velocity profiles in the ESP were far from uniform, and following discussion with the company the AFDL was commissioned to undertake a small-scale physical model study of the flows in the ESP. The aim of the project was to obtain uniform velocity profiles. Water was used as the working fluid for the flow visualisation study. This had two distinct advantages:

- the kinematic viscosity is 15 times lower in water than it is in air, thus allowing the flow velocities to be 15 times slower in water for the same Reynolds number; and
- it is much easier to undertake flow visualisation studies in liquids.

A 1:30 scale model of the ESP was constructed from acrylic (Figure 3). Flow visualisation studies confirmed that the modifications suggested during the upgrades were detrimental to the generation of uniform flows in the ESP. These studies also showed that by removing the modifications fitted during the upgrades and redesigning the inlet perforated plate wall, a near uniform flow velocity was generated through the ESP. When these modifications were fitted in the refinery ESP, the solids loading in the exhaust stack was reduced by an order of magnitude.

The cost to the company of the small-scale modelling undertaken at the AFDL was \$90K and was a major saving in capital cost for the unit to satisfy statutory emission licence conditions.

5.0 SUMMARY

The four examples described above show what can be achieved when people in industry collaborate closely with people in CSIRO.

For a successful collaboration, it is essential that the CSIRO researchers, who have the facilities and expertise necessary to undertake a particular collaborative exercise, focus on the customer's needs and that the customer participates in the research being undertaken.

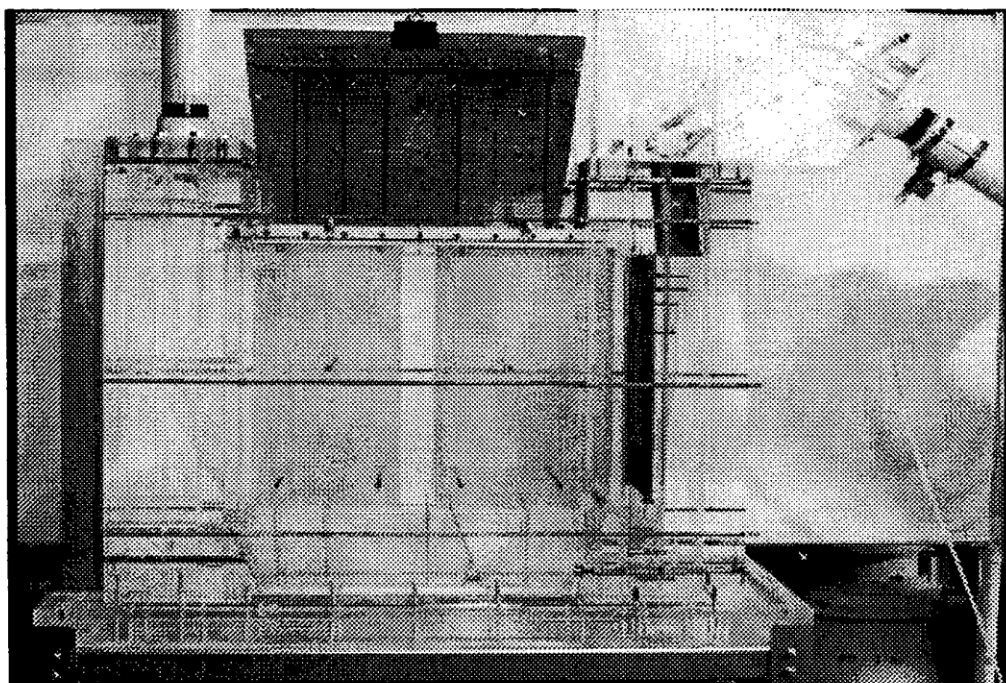


Figure 3
Photograph of the 1:30 acrylic scale model of the ESP

6.0 CONCLUSIONS

From the examples described above it is concluded that small-scale physical modelling and flow visualisation have a very important role to play in cost-effective solutions to problems associated with the flow of gases, gas/solids and liquid/solids in the mineral processing industry. The benefit to research cost ratio can be as high as 30 after one year's operation following the modification of plant in accordance with the recommendations of the research.

It is concluded that computational fluid dynamics, used in conjunction with small-scale physical modelling, can be the most cost-effective method of providing robust solutions to some problems encountered in the mineral processing industry.

It is also concluded that a successful collaboration between industry and CSIRO is critically dependent on there being close communications between the industry personnel and the CSIRO researchers throughout the project.

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APPENDIX A**NON-DIMENSIONAL ANALYSIS OF THE FLUID DYNAMICS OF SUSPENSIONS****A1.0 NOTATION**

B	Bingham number	—
C	colloid number	—
Cd	drag coefficient	—
c_v	volumetric concentration	—
D	fluid diffusivity	$m^2 s^{-1}$
Dp	particle diffusivity	$m^2 s^{-1}$
d	diameter of particle	m
F	force	N
F	momentum inverse relaxation time	s^{-1}
Fr	Froude number	—
g	gravitational constant	$m s^{-2}$
He	Hedstrom number	—
k	shape factor	—
l	characteristic length	m
Pl	plasticity number	—
Re	Reynolds number	—
Re'	generalised Reynolds number	—
St	Stokes number	—
t	time	s
V	local mean velocity	$m s^{-1}$
V'	perturbation velocity	$m s^{-1}$
V_∞	terminal settling velocity	$m s^{-1}$
ϵ	eddy length scale	m
γ	shear rate	s^{-1}
κ	wave length	m
κ	Boltzman constant	$J/^\circ K$
μ	dynamic viscosity	$Pa.s$
μ_s	coefficient of sliding friction	—
η_B	Bingham plastic viscosity	$Pa.s$
τ_y	yield stress	$Pa.s$
ρ	density	$kg m^{-3}$
ρ^*	density ratio	—

Subscripts

f	fluid	—
m	mixture	—
s	solid	—

A2.0 SUSPENSION PROPERTIES**A2.1 General**

A suspension's characteristics are primarily influenced by particle size, density and chemistry of the system. For very small particles in liquid, say $d < 2$ mm, the suspension is often colloidal and may be considered to be a homogenous fluid. For larger particles, say $d < 100$ mm, the particles may combine with the liquid to form flocculated suspensions or gels, which may also be considered homogenous. For even larger particles or for gas/solid suspensions, the particles generally behave as a distinct phase and interaction with the fluid occurs through some form of inter-phase friction.

In practice, the wide particle size or density distributions used means that hybrid behaviour incorporating several modes often prevails. However, only the homogenous and heterogeneous cases will be considered here.

A2.2 Homogenous Fluids

For colloidal or flocculated suspensions it is necessary to obtain the homogenous fluid's rheology, which is generally non-Newtonian. Depending upon the type, this fluid can then be described with various numbers, e.g.,

$$B = \frac{\tau_y d}{V \eta_B}, \quad He = \frac{\tau_y d^2 \rho_f}{\eta_B^2}, \quad Pl = \frac{\tau_y}{\rho_f V^2} \quad \text{or} \quad C = \frac{\mu \gamma d^3}{\kappa T} \quad (1)$$

which describe the fluid structure, while the fluids flow properties can be described using a generalised Reynolds number,

$$Re' = \frac{\rho_f V l}{\mu_{app}} \quad (2)$$

where μ_{app} is the apparent viscosity at the shear rate of concern and embodies the rheological behaviour of the fluid.

In this way the flow behaviour of the suspension can be modelled using similar techniques used for Newtonian fluids, i.e. gas or water. However, this is not a trivial exercise and complete similitude is seldom achieved.

A2.3 Heterogeneous Suspensions

Given the difficulty in obtaining similitude for homogenous suspensions, it is not surprising to discover that complete similarity cannot be obtained for the more complex heterogeneous suspension. However, insight can be gained by considering the various components of the flow.

Suspension flow in any reactor or process vessel may be approximately grouped into three areas:

- response of discrete particles to changes in fluid flows, e.g. changes in direction or turbulent fluctuations, i.e. the dynamic response;
- the overall flow patterns of the suspension within the vessel, or macro flow patterns; and
- the formation and stability of stratified flows or interfaces.

A2.3.1 Dynamic response

For a single particle in 1-D creeping flow in the absence of any field forces, the force balance on an unconstrained dilute suspension of non-interacting particles that are sufficiently massive for Brownian motion effects to be ignored, may be approximated by

$$\rho_s \frac{\pi d^3}{6} \frac{dV_s}{dt} = \rho_s \frac{\pi d^3}{6} \left(\frac{k_s}{\rho^*} \frac{d(V_f - V_s)}{dt} + F(V_f - V_s) \right) \quad (3)$$

or

$$F_i = F_v + F_d \quad (4)$$

where the first term on the right-hand side is due to the virtual mass, or added mass associated with the particle, the second term is the drag force, ρ^* is the density ratio $\rho^* = r_s/r_f$, k_s is a shape factor ($= 0.5$ for a spherical particle), and F is the inverse momentum relaxation time defined as

$$F = \frac{3Cd|V_f - V_s|}{4\rho^* d} \quad (5)$$

For dynamic similitude the ratio of the forces must remain constant, hence

$$F_d/F_i = IF(V_f - V_s)/V_s^2 = Ft(V_f - V_s)/V_s \quad (6)$$

must remain constant, where t is a characteristic time.

F is an implicit function of the particle's Reynolds number and matching it for all particle velocities would be a formidable task. However examination of the particle's terminal settling velocity V_∞ provides a very similar formulation, namely

$$V_\infty = \left(\frac{4gd(\rho^* - 1)}{3Cd} \right)^{0.5} \quad (7)$$

These equations differ only slightly and replacement of (5) with a function of V_∞ and ρ^* provides a very close approximation to the characteristics of a particle's response to a change in velocity. This is especially so when one considers that (a) particle accelerations in most processes will be of the order of $1g$, and (b) slip velocities have been shown by various authors (Richardson and Zaki, 1954; Tsuji *et al.*, 1984; Pullum, 1980) to be comparable to V_∞ in particles undergoing vertical or horizontal transport.

The use of the particle's settling velocity has the advantage that it can be measured or calculated (Haider and Levenspiel, 1989) and provides a very compact encapsulation of the particle's response characteristics.

Other particles in suspension modify the movement of individual particles through interaction of their wakes or through collision. This 'hindering' of individual particles is a function of particle volumetric concentration (Shook and Roco, 1991) and (3) should be modified to include this, viz.

$$\rho_s \frac{\pi d^3}{6} \frac{dV_s}{dt} = \rho_s \frac{\pi d^3}{6} \left(\frac{k_s}{\rho^*} \left(\frac{1+2c_v}{1-c_v} \right) \frac{d(V_f - V_s)}{dt} + \frac{F}{(1-c_v)^{1.7}} (V_f - V_s) \right) \quad (8)$$

Similarity thus requires concentration to be maintained as well.

Considering turbulent suspension flows the problem of scaling becomes far more complex. It is generally accepted that the turbulent velocity fluctuations of the carrier fluid are characterised by an energy spectrum which is modified by the presence of the particles. This energy spectrum can be divided into three regions: the production region where the eddies are large, $\varepsilon \gg d$, the transfer region where the eddies are comparable in size to the particles, $\varepsilon \approx d$, and the dissipation region where the particles are normally larger than the eddy length scales, $d > \varepsilon$. In the production region, the particles are normally entrained within the eddy and generally dissipate some of the energy, while in the dissipation region the particles do not significantly alter the turbulent structure, although in some cases they can increase turbulence through vortex shedding.

It can be shown (Roco, 1993) that for the large eddies in the production zone the particle perturbation velocity

$$V_s^l = \varphi(\rho^*, St) \quad (9)$$

where St is the Stokes number, i.e. the ratio of the particle and flow characteristic times

$$St = \left(\frac{\mu}{\rho_f V_f^l \kappa d^2} \right)^2 \quad (10)$$

and $u_f^l \kappa$ is the circular frequency of the fluid motion (note: for particles to be contained within the eddy $St < 1$).

Similarly, for small-scale eddies

$$V_s^l = \varphi(\rho^*, \varepsilon/d) \quad (11)$$

Motion for intermediary eddies is obtained through interpolation.

It is very unlikely that matching a particle size distribution and turbulent spectra with full-size conditions can ever be achieved physically. The eddy sizes range from the largest flow dimension to the smallest determined by the rheology of the fluid – the Kolmogorov micro-scale. If the fluid in the model and the full-size system have comparable rheologies, then scaling will change the larger eddies but maintain the micro-scale. Suitably scaling the particle size distribution may maintain particle-eddy interactions in the large eddies, but the inevitable introduction of ultra-fine particles will increase the 'effective fluid' viscosity and increase the micro-scale, further reducing the range of ε/d in the model. Attempts to model scaling effects of suspension interaction at this level are better handled computationally where once the behaviour has been observed on the small scale, extension to the full size can be done through appropriate multiphase computational fluid dynamics.

It is clear that detailed attempts to physically scale model the interaction of a suspension of particles with a turbulent fluid is unrealistic. However, individual particles respond to each turbulent fluctuation in a manner similar to that described by (3) and suitable scaling

correlations can be found expressed in terms of particle diffusivity, Dp , F and ρ^* (Soo, 1989). For very small particles with no rheological action $Dp \rightarrow D$, the fluid diffusivity, which is a function of Reynolds number, but as the particle size or mass increases, Dp becomes a function of F , c_v and the turbulent spectra again. Since it is not possible to obtain similitude of the turbulent spectra, one is left with the reduced aim of providing similitude in the manner to which particles respond to a fluid perturbation which, from the above analysis, requires that V_∞ , ρ^* and c_v are maintained.

A2.3.2 Macro flow patterns

The difficulty in obtaining complete similitude in the relatively simple case of a suspension dispersed in an unconstrained turbulent flow field demonstrates that complete similarity will not be possible for a constrained flow inside a vessel. However, at a scale of scrutiny comparable to the process vessel's dimensions, suspension flows are often quite uniform especially for well mixed flows. The suspension within the vessel may be considered to behave as a homogenous fluid having the bulk properties of the suspension. Using such an approach dynamic similarity is obtained by maintaining a Reynolds number as before.

A2.3.3 Interfaces

In cases where the suspension is not so well mixed concentration gradients and/or regions that vary markedly in particle content are formed. Average fluid properties are inappropriate and local values must be used. These concentration gradients normally occur through external body forces, e.g. gravity, and inclusion of these terms is necessary. Consider, as an example, the case of the suspension of the solids in a mixing tank. Despite the large number of variables involved in describing a heterogeneous suspension, considerable success has been achieved by considering very few dimensionless groups. Rose and Duckworth (1969), for example, demonstrated that for narrow size distributions of non-interacting particles, results obtained for conveying a very wide range of particle densities, diameters and concentrations suspended in either air or water could be grouped together purely in terms of the pipe Froude number V^2/gD .

The functional relationship they developed demonstrated that despite the density ratio range of three decades and a diameter ratio range of two decades, the fundamental behaviour of the suspensions were similar. Their correlation was obtained for suspensions transported along pipes with conveying velocities well in excess of that required for incipient motion. Furthermore most of the data was obtained with pneumatic systems where the density ratio is >1000 . For the more normal case of solids in liquid, the buoyancy forces on the particle must be taken into account and a better correlating parameter is a modified Froude number which includes these buoyancy effects, namely

$$Fr = V^2 / (g(\rho^* - 1)) \quad (12)$$

Many correlations have exploited this group, e.g. Buurmann *et al.* (1985) demonstrated that the non-dimensional height of the suspension in a mixing tank was well described using this group and a diameter ratio, although generally this later group becomes progressively unimportant as vessel size increases.

The Froude number is also used to characterise the waviness of interface surfaces – its original use. Consequently, by maintaining the modified Froude number, dynamic similitude is obtained for both the suspending characteristics of the flow, i.e. the extent of a concentration gradient or stratified layer, and the stability of this layer due to gravitational waves etc.

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